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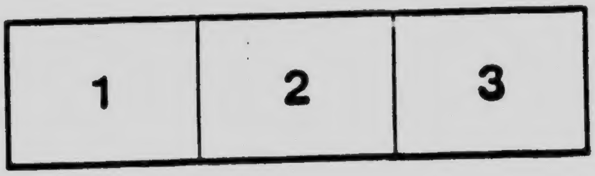

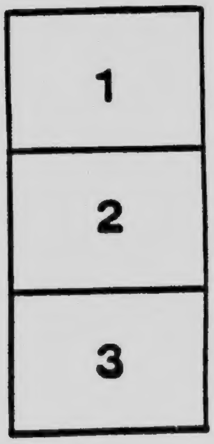

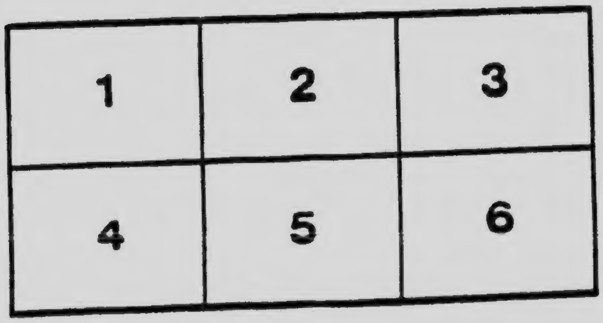

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THE SPECTROSCOPIC ORBITS OF THE ECLIPSING VARIABLES U OPHIUCHI, RS VULPECULAE, AND TW DRACONIS

BY J. S. PLASKETT

INTRODUCTION

In determining the orbital elements of a spectroscopic binary from a series of measurements of the radial velocity, we can usually obtain the period, the eccentricity of the relative orbit, the maximum orbital velocity, the velocity of the centre of mass of the system, and the longitude and time of periastron.

The inclination of the plane of the orbit to the plane tangent to the celestial sphere is indeterminable and consequently, though formulæ are available giving the semi-axis major and the mass, these both appear as functions of the sine of the inclination, and their actual values must remain unknown so long as the inclination is unknown. Further, in respect to the expression for the mass of the system, it appears as a function of the ratio of the masses of the two bodies and unless this is known, which is the case only in the comparatively few instances when the spectra of both components are present, neither the mass of each nor the total mass of the system, always appearing with the function of the inclination attached, can be determined.

It is only in two special cases that the inclination can be obtained: in visual binaries whose orbital elements have been determined and in eclipsing (Algol) variables of which the light curve is accurately known.

In the case of visual binaries, with the inclination and the relative radial velocities of the two components known, we can evidently obtain the masses, the dimensions of the orbit, and the parallax, but not the diameter and densities of the two stars.

In the other case, the one under consideration, where the spectroscopic binary revolves in a plane so nearly in the line of sight that the stars mutually eclipse one another twice in the period, then, if an accurate light curve is available, not only the inclination but the relative dimensions of the two bodies in terms of the major axis of the system can be determined. When, in addition, both spectra can be measured, complete information can be obtained about the actual diameters, masses, densities, and distance apart of the two bodies. This is of importance in view of the small number of stars for which we have such data.

In the three orbits to be discussed here both spectra have been observed in U Ophiuchi and R S Vulpeculæ, in which consequently complete dimensions are available. With T W Draconis the second spectrum is invisible and the dimensions in this case must depend upon an assumption as to the ratio of the masses of the two bodies.

U Ophiuchi

The eclipsing variable U Ophiuchi (α 17h 11·4m; δ + 1° 19′ 1900, vis. mag. 5·7, spectral type B9) was placed under observation for radial velocity on Mar. 19, 1919, and observations were continued until June 4. In all 18 plates were obtained of which 14 were used in determining the spectroscopic elements. The remaining four plates were so near the minimum or zero velocity in the orbit that the doubled lines could not be separated, with the result that their velocities are unreliable and were not used.

The lines of the spectrum are rather wide and are diffuse and faint, lacking in contrast, this latter probably due to the superposition of the continuous spectrum of the one star on the absorption lines of the other. One plate, No. 1983, obtained at principal minimum shows the spectrum with single lines. This spectrum is principally, but not wholly, as it is only a partial eclipse, of the fainter star, and hence gives a truer idea of the character and type. From this spectrum the type should be classed as B5, but is rather abnormal in the breadth of the hydrogen lines. In all 9 double lines have been measured Hγ, Hδ; He 4472, 4388, 4144, 4026; C 4267; Mg 4481; Ca. K. 3934. Owing to the lack of contrast and the diffuseness of the lines, the measures are difficult and necessarily not of very high accuracy. In many cases some of the lines of the weaker spectrum could only be recognized and bisected by their showing a faint lightening of the spectrum on each side of the wire. Nevertheless, I believe the measures are reliable, as there is fair interagreement among the values for the different lines, and the general dimensions of the system are substantially correct.

TABLE I—OBSERVATIONS OF U OPHIUCHI

Plate Number	Date	Julian Date	Phase	Velocity		No. of Lines	Residuals O–C	
				Brighter	Fainter		Brighter	Fainter
	1919							
1745	April 2	2,051·022	0·362	−196·9	+179·3	8	− 9·7	− 9·1
1776	" 7	2,056·974	1·282	+148·4	−222·6	8	−19·2	− 7·3
1805	" 13	2,062·973	0·572	−156·7	+159·3	7	+ 6·0	− 1·2
1808	" 14	2,063·019	0·618	−148·2	+128·8	9	− 4·4	−10·2
1905	" 28	2,077·935	0·438	−174·7	+215·4	8	+16·2	+22·8
1911	" 29	2·078·972	1·475	+139·4	−150·2	8	+24·7	+ 4·5
1935	May 3	2,082·876	0·347	−194·2	+194·2	8	− 9·5	+ 8·6
1952	" 4	2,083·866	1·337	+149·0	−194·8	8	−12·3	+13·3
1983	" 6	2,085·885	0·000		− 26·1	6	+14·6	+14·6
2009	" 19	2,098·884	1·259	+181·7	−216·2	8	+13·4	− 0·1
2053	" 30	2,109·911	0·544	−170·4	+174·3	6	+ 1·6	+ 3·1
2072	June 2	2,112·874	0·152	−118·4	+ 73·7	6	− 7·3	−28·1
2084	" 3	2,113·870	1·148	+155·1	−201·1	8	+ 1·9	− 2·1
2097	" 4	2,114·880	0·481	−187·8	+193·8	6	− 1·3	+ 6·1

In the above table the first three columns give the plate number, the date and Julian date of the observations. The fourth column gives the phase from primary minimum of the photometric orbit computed from the initial phase J. D. 2,418,026·703 with the period 1·6773476 days. The fifth and sixth columns give the observed velocities and the eighth and ninth the residuals from the final orbit of the brighter and fainter components respectively, while the seventh column contains the number of doubled lines measured on each plate, with the exception of plate 1983, on which six single lines were measured.

As the photometric orbit does not show any ellipticity and as the observations seem to follow sine curves as closely as can be expected, the orbit was assumed to be circular and consequently only K, the half amplitude and γ the velocity of the system, remain to be determined. From smooth curves drawn through the observations K_b was assumed 182·0 km., K_f 210·0 km. and γ −12·0 km.

The velocity at any phase θ is given by $V = \gamma + K \sin \theta$ and to apply least squares corrections we have $\delta V = \delta \gamma + \delta K \sin \theta$. An ephemeris and observation equations were formed, resulting in the normal equations

$$\begin{aligned} 26{\cdot}0\,\delta\gamma - 2{\cdot}361\,\delta K_b + 2{\cdot}361\,\delta K_f - 5{\cdot}7 &= 0 \\ 10{\cdot}424\,\delta K_b \qquad\qquad + 24{\cdot}38 &= 0 \\ 10{\cdot}424\,\delta K_f + 55{\cdot}09 &= 0 \end{aligned}$$

whose solution gave

$$\delta\gamma = +0{\cdot}5 \qquad \delta K_b = -2{\cdot}2 \qquad \delta K_f = -5{\cdot}4$$

and the final elements

$$\begin{aligned} K_b &= 179{\cdot}8 \pm 2{\cdot}70 \text{ km.} \\ K_f &= 204{\cdot}6 \pm 2{\cdot}70 \text{ km.} \\ \gamma &= -11{\cdot}5 \pm 1{\cdot}88 \text{ km.} \end{aligned}$$

while the probable error of a single plate for each spectrum was the same and equal to ± 8·3 km. Considering the character of the spectrum and the difficulty of measurement these results a[illegible]y, the uncertainty of K and consequently of the dimensions of the system [illegible] slightly over one per cent.

Applying th[illegible]ues by the well-known formulae, we have

$$\begin{aligned} a_b \sin i &= 4{,}147{,}000 \text{ km.} \\ a_f \sin i &= 4{,}718{,}000 \text{ km.} \\ (m_b + m_f) \sin^3 i &= 9{\cdot}890\ \odot \end{aligned}$$

In Shapley's photometric orbit* three solutions are obtained, two assuming uniformly illuminated discs and a third postulating darkening towards the limb. Shapley's second and third solutions are applied here, one uniform, one darkened. These give inclinations of 85° 42′ and 83° 58′ respectively, and make the radius of each star 0·252 of the semi axis of the relative orbit in each solution. The dimensions, masses, and densities of the system at once follow.

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*Contributions from the Princeton Observatory No. 3, p. 84.

000257[illegible]9

	Uniform	Darkened
a semi axis relative orbit	8,800,000	8,915,000
a_b " brighter star	4,158,000	4,170,000
a in terms of radius of sun..	12·78	12·82
a_b " "	5·84	5·86
r_b radius brighter star	3·22	3·23
r_f " fainter "	3·22	3·23
m_b mass brighter star	5·31	5·36
m_f " fainter "	4·66	4·71
ρ_b density brighter star	0·20	0·18
ρ_f " fainter "	0·18	0·16

The volumes of the stars are computed assuming the elliptical forms given by the photometric orbit and

volume brighter star	26·28 ⊙	29·07 ⊙
volume fainter star	26·28	29·07
and the total surface area of both stars	18·88	19·62

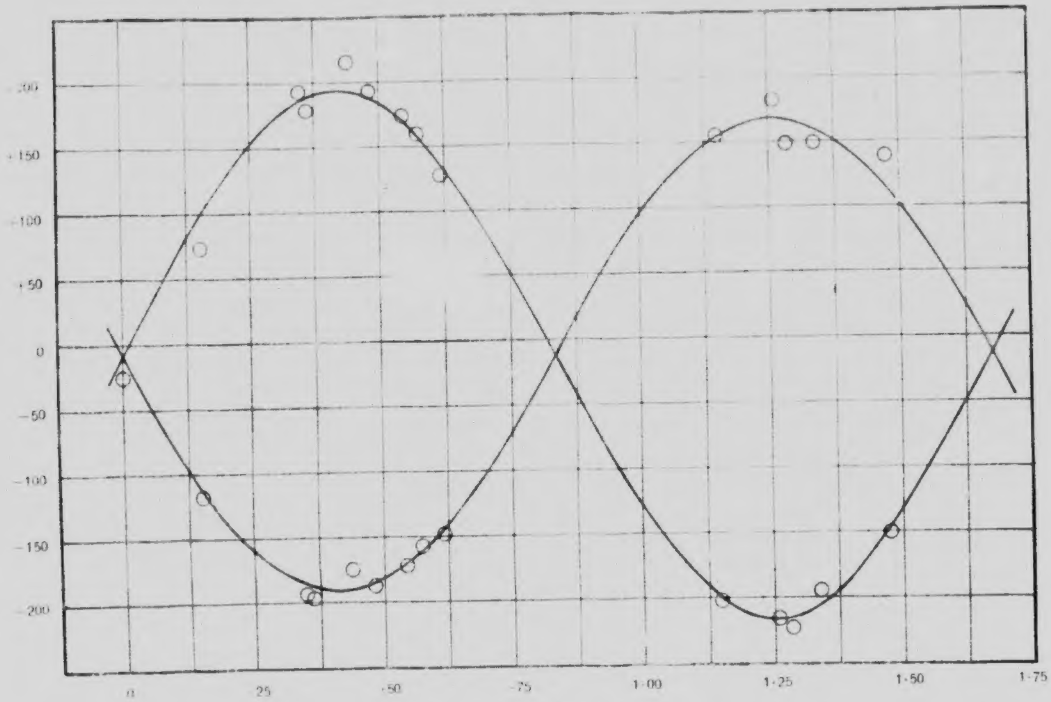

Fig. 1. U Ophiuchi

If we assume the surface intensity of a B5 star to be −2·5 magnitudes,[1] the sun's absolute magnitude to be 4·86, the absolute magnitude of the brighter component of

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[1] Astrophysical Journal 40, p. 415, 1914.

U Ophiuchi whose apparent magnitude is 6·35, is −0·19, and hence the parallax is 0″·0049.

A graph showing the sine curves for the two components with the velocities represented by open circles is shown in Fig. 1.

R S Vulpeculæ

The eclipsing variable R S Vulpeculæ (α 19h 13·4m, δ +22° 16′, 1900, type A, vis. mag. 7·30) was placed under observation April 26, 1919, and the last plate used in the principal orbit was obtained on July 20, 1919. A fine grained Seed 23 plate was obtained on July 30 and used for the second spectrum. Fourteen plates were obtained in this interval, all of which have been measured and used. The spectrum is of type B8 instead of A, and the magnitude as judged by comparison with U Coronæ and T W Draconis appears considerably brighter than given by Nijland and Stewart.

This binary is especially interesting on account of the great disparity in size of the two components, the fainter star being five times the diameter of the brighter and nine-tenths as bright. It would consequently be expected, when the two stars are of nearly equal brightness, that the second spectrum would be plainly visible, but it can only be seen and measured with great difficulty, and it is estimated to be of only about one-fourth the intensity of the brighter spectrum. Nevertheless, it was measured on six plates, and although the residuals are in some cases rather large, the probable error of the measures of the second spectrum on a single plate being ±8·1 km., the lines and plates are in fair interagreement. Consequently, there can be no doubt of the reality of the second spectrum, even though the mass of the fainter body is only 0·31 that of the brighter, a greater difference in masses when both spectra appear than has previously been found. Why the second spectrum should be so relatively faint when the two stars are of nearly equal brightness is not apparent. It may be that, although the continuous part of the spectra are of nearly equal intensity, the absorption lines of the second body are fainter than those of the primary rendering them difficult to see when the continuous spectrum of the primary is superposed. Or, again, a more likely explanation is that the lines are widened so much by the rotation of the large diameter faint star as to be made relatively very faint. One plate was made about 2·[illegible] hours after primary minimum, which, according to Stewart's orbit*, would be about an [illegible] after the total phase, so that although it would receive most of the light from the fainter component, about one-fifth would come from the brighter. This spectrum has much the same character as the others, except that the lines are much weaker and thus is in agreement with the above hypothesis. Further an additional plate was obtained on July 30 on Seed 23 emulsion and the finer grain enables the second spectrum to be more readily and certainly measured. On this plate the enhanced line 4549 is plainly doubled, the intensity of the second spectrum being relatively much stronger than in the other lines. Further, the silicon pair 4128, 4131 show fairly strong companions, while the second spectrum in the hydrogen and helium lines is very weak. This would make it appear as if the faint diffuse companion were of Type B9 with relatively weak hydrogen and helium lines, while the bright dense star is B8 or even earlier. The relative intensities of the doubled lines 4549, 4131, 4128 are more nearly equal being only about one-half as compared with one-fourth for the hydrogen and helium lines.

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*Astrophysical Journal, 42, 315, 1915.

Altogether 11 lines have been measured in the primary spectrum, Hγ, Hδ; He 4472, 4388, 4144, 4121, 4026; Fe-Ti 4549; Mg 4481; C 4267; Ca. K. 3934 and in the fine grained plate Si 4128, 4131. The lines are of only fair quality for measurement, although much better defined than in U Ophiuchi, but the measures nevertheless are satisfactorily accordant, the probable error of measures of the primary spectrum on a plate being only ± 1·8 km. per second.

In the table of observations given below column 1 contains the plate number, columns 2, 3, date and Julian date of the observation, and column 4 the phase from primary minimum computed from the original phase 2,419,652·963 with a period of 4·477325 days from Stewart's photometric orbit.* Columns 5, 6 contain the velocities of the primary and secondary stars, and columns 8, 9 the residuals in the sense observed minus computed from the final orbit. Columns 7 contains the number of lines measured in the primary, and where a second figure is present the number in the secondary spectrum.

TABLE II OBSERVATIONS OF R S VULPECULAE

Plate Number	Date	Julian Date	Phase	Velocity		No. of Lines		Residuals O-C	
				Brighter	Fainter			Brighter	Fainter
	1919								
1888	April 26.....	2,075·981	0·786	−68·3	+132·0	9	10	−2·30	+13·2
1912	" 29.......	078·988	3·793	+22·2		11		−1·28	
1987	May 6.....	085·969	1·818	−63·2	+107·4	8	4	−3·20	+ 7·8
2243	July 1....	141·903	4·024	+11·2		10		−2·11	
2269	" 3.......	143·903	1·547	−70·7	+131·9	9	7	+2·98	−11·5
2298	" 7.......	147·883	1·050	−76·9	+160·8	9	8	−0·99	+10·3
2316	" 8.......	148·902	2·069	−39·1		7		+2·36	
2329	" 9.......	149·870	3·037	+27·5	+163·7	6	3	+1·96	−10·3
2350	" 13.......	153·818	2·507	− 7·5		10		−2·27	
2367	" 14.......	154·773	3·461	+28·2		7		−2·79	
2390	" 15.......	155·878	0·090	−22·9		6		−1·91	
2428	" 18.......	158·897	3·109	+29·8		10		+2·07	
2441	" 19.......	159·892	4·104	+14·1		9		+5·08	
2449	" 22.......	160·727	0·462	−44·4		6		+2·56	
2579	" 30.......	170·760	1·540	−67·2	+132·2	9	8		−11·6

When these velocities and phases were plotted on cross section paper, it was at once seen that the orbit was not circular, although the photometric orbit does not show eccentricity, but this element can only rarely be obtained with accuracy from the photometric observations. Preliminary elements obtained graphically were assumed as follows

e	eccentricity............	0·05
K	half amplitude velocity	54·0 km.
γ	velocity of system............	−22·65 km.
ω	longitude of apse............	240°
T	time of periastron............	1·903 days from minimum.

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*Astrophysical Journal, 42, 315, 1915.

An ephemeris and observation equations calculated by means of Lehman Filhés differential coefficients were computed for applying least squares corrections to e, K, γ and ω. Owing to the smallness of the eccentricity it was considered useless to apply corrections for both ω and T and the latter was considered fixed. The observation equations are given in the following table, where x, y, z, u have the values

$$x = \delta\gamma$$
$$y = \delta K$$
$$z = K\delta e$$
$$u = K\delta\omega$$

TABLE III. OBSERVATION EQUATIONS RS VULPECULAE

1	1·000x	− ·288y	+ ·004z	+ 1·008u	+ 0·91 = 0
2	1·000	+ ·366	+ ·996	+ ·964	+ 1·63
3	1·000	+ ·815	+ ·279	+ ·436	− 1·83
4	1·000	+ ·929	+ ·082	+ ·342	− 2·27
5	1·000	+ ·960	− ·758	− ·127	+ 1·02
6	1·000	+ ·796	− ·978	− ·523	− 1·71
7	1·000	+ ·598	− ·728	− ·738	− 1·53
8	1·000	+ ·576	− ·579	− ·798	− 8·80
9	1·000	− ·045	+ ·511	− ·956	− 2·17
10	1·000	− ·511	+ ·984	− ·830	− 5·86
11	1·000	− ·838	+ ·658	− ·538	+ ·37
12	1·000	− ·995	− ·029	− ·199	+ 0·52
13	1·000	− ·904	− 1·010	+ ·519	− 0·78
14	1·000	·635	− ·715	+ ·836	+ 6·25

From these observation equations the following normals were obtained:

$$14{\cdot}000x + 0{\cdot}847y - 1{\cdot}279z - 0{\cdot}606u - 11{\cdot}34 = 0$$
$$7{\cdot}240 - 1{\cdot}234 - 0{\cdot}506 - 9{\cdot}202 = 0$$
$$6{\cdot}670 - 0{\cdot}058 + 0{\cdot}744 = 0$$
$$6{\cdot}624 + 24{\cdot}250 = 0$$

Their solution gave

$x = +0{\cdot}61$ $\delta\gamma = +0{\cdot}61$ $\pm 0{\cdot}56$
$y = +0{\cdot}98$ $\delta K = +0{\cdot}98$ $\pm 0{\cdot}79$
$z = +0{\cdot}157$ $\delta e = +0{\cdot}0029$ $\pm {\cdot}0149$
$u = -3{\cdot}530$ $\delta\omega = -3^\circ{\cdot}74$ $\pm 0{\cdot}84$

whence the final elements

$e = 0{\cdot}053 \pm {\cdot}015$
$K_b = 54{\cdot}98 \pm 0{\cdot}79$ $K_f = 175{\cdot}9$
$\gamma = -22{\cdot}04 \pm 0{\cdot}56$
$\omega = 236{\cdot}26^\circ \pm 0{\cdot}84$
$T = 1{\cdot}903$ days

The probable error of a single plate for the bright star is ± 1·8 km, and for the faint ±8·1 km per second. A graph of the orbit with the observations of the principal spectrum represented by circles is shown in Fig. 2.

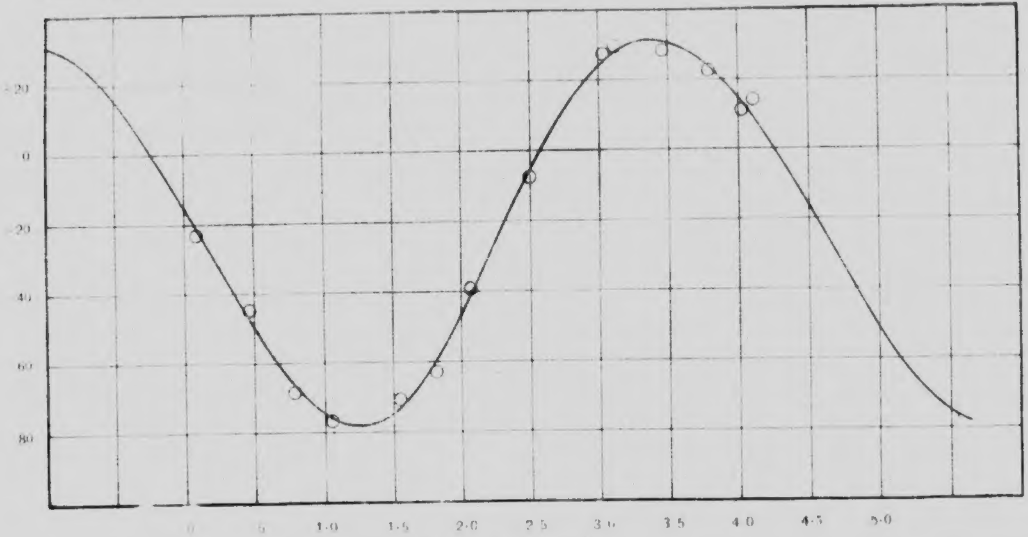

Fig. 2. R S Vulpeculæ

Applying the above values we obtain by the well known formulæ

$a_b \sin i = 3{,}388{,}000$ km.

$a_f \sin i = 10{,}842{,}000$ km.

$(m_1+m_2)\sin^3 i = 5\cdot70\odot$

Stewart's photometric orbit* gives two solutions, a uniform, and a darkened. The uniform gives a value of the inclination 69° 45′, radius brighter star ·090, radius fainter star ·450, while the darkened solution gives 68° 25′, ·0932, ·466 for these three quantities. The dimensions, masses and densities in the orbit follow. For comparisons the dimensions from Shapley's darkened orbit† are also given and it appears that further photometric work on this star is desirable.

	Uniform	Darkened	Shapley
a semi axis relative orbit	15,490,000	15,300,000	14,230,000
a_b " orbit brighter star	3,610,000	3,644,000	3,389,000
a in terms of sun's radius	21·84	22·00	20·47
a_b " "	5·19	5·24	4·88
r_b radius brighter star	1·96	2·05	1·80
r_f " fainter "	9·81	10·25	5·16
m_b mass brighter "	5·26	5·40	4·34
m_f " fainter "	1·64	1·69	1·36
M total mass	6·90	7·09	5·70
ρ_b density brighter star	0·70	0·63	0·74
ρ_f " fainter "	0·0017	0·0016	0·008

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*Astrophysical Journal 42, 315, 1915

†Contributions from Princeton Observatory, No. 3, p. 90

If we assume the surface intensity of a B8 star to be −2·2* magnitudes and the sun's absolute magnitude to be 4·86 the absolute magnitude of the brighter component of R S Vulpeculæ, with apparent magnitude 7·75, is +1·09 and the parallax is 0″·0046.

T W Draconis

The eclipsing variable T W Draconis (α 15h 32·4m., δ+64° 14′, 1900; vis. mag. 7·45; spectral type B9) was first observed on April 13, 1919, and the last plate obtained on July 17. During this interval 14 plates were obtained, all of which are used in the orbit. They are not so well distributed as in R S Vulpeculæ, owing to the depth of primary minimum, when the star falls to 9·8 magnitude, making it impracticable to obtain plates at this epoch. The comparative faintness of the secondary, its light being 0·115 of the system, of course renders its spectrum unobservable.

The spectrum classed as B9 is in reality A3 judging by the relative strength of hydrogen and K and the number and intensity of the metallic lines, and in consequence should be capable of accurate measurement. Although the interagreement among the lines is fairly good, the velocities of the plates are disappointing, the residuals unexpectedly high, resulting in a plate error of ±2·6 km per second. This is probably due to the rather wide and diffuse character of the metallic lines, making the settings uncertain. That the lines are wide should not be a cause for surprise when we consider the nearness of the two bodies and the probable rapid rotation of the bright body in the period of the system.

The data of the observations are given in Table IV, where column 1 contains the number of the plate, columns 2 and 3 the date and Julian date of the observation. Column 4 contains the phase from primary minimum computed from initial photometric phase 2,418,906·453 with period 2·80654 days and column 5 the velocity determined from measures of the number of lines in column 6. Column 7 contains the residuals in the sense observed minus computed from the final orbit.

TABLE IV—OBSERVATIONS OF T W DRACONIS

Plate Number	Date	Julian Date	Phase	Velocity	No. of Lines	Residuals O-C
	1919					
1802	April 13	2,422,062·890	1·886	+50·8	12	−2·33
1883	" 26	075·862	0·825	−63·2	20	−0·76
1909	" 29	078·922	1·078	−38·7	17	+3·99
1936	May 3	082·840	2·190	+62·2	12	−3·19
1982	" 6	085·865	2·408	+51·4	12	−4·47
2217	June 30	140·734	1·147	−37·3	17	−2·16
2233	July 1	141·721	2·134	+67·6	11	+2·40
2247	" 2	142·716	0·322	−47·9	7	−2·74
2275	" 6	146·733	1·532	+18·6	13	+4·53
2289	" 7	147·726	2·525	+50·3	11	+6·13
2306	" 8	148·726	0·719	−61·4	14	+4·37
2363	" 14	154·718	1·098	−48·1	15	−7·44
2383	" 15	155·722	2·102	+65·1	9	+0·45
2398	" 17	157·717	1·290	−16·4	11	+1·29

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*Astrophysical Journal 40, 415, 1914.

When these observations were plotted it was at once seen that by making the orbit slightly eccentric better agreement could be obtained than with a circular orbit. Although no eccentricity is given by Shapley's orbit,* it is probable that Nijland's observations are not of sufficient accuracy to determine this, and alternative solutions for eccentric and circular orbits gave considerably lower residuals for the former. Preliminary elements which were obtained graphically are as follows.

Period from photometric orbit	2·80654 days
Eccentricity e	0·04
Semi-amplitude K.	64·5 km.
Velocity of system γ	−0·5 km.
Longitude of apse ω	90°
Time of periastron T.	0·020 days

Owing to the smallness of the eccentricity it was considered useless to apply least squares corrections to both T and ω and, in this case, as the latter seemed better determined by the graphical elements, a correction for T was used. The differential coefficients obtained by Lehman Filhés were used in computing an ephemeris and observation equations which are given in Table V.

TABLE V OBSERVATION EQUATIONS OF TW DRACONIS

1.	1·000x	− ·666y	−1·010z	+ ·792u	+ 4·47 = 0
2.	1·000	− ·997	+ ·147	− ·073	− 3·43
3	1·000	− ·953	+ ·576	− ·296	+ 1·24
4.	1·000	− ·659	+ ·978	− ·708	− 4·29
5.	1·000	− ·626	+ ·963	− ·732	+ 7·22
6	1·000	− ·543	+ ·899	− ·784	+ 1·78
7.	1·000	− ·272	+ ·515	− ·889	− 1·68
8.	1·000	+ ·223	− ·427	− ·901	− 4·72
9	1·000	+ ·824	− ·925	− ·541	+ 1·86
10.	1·000	+ ·991	− ·258	− ·129	− 1·65
11.	1·000	+ ·998	− ·119	− ·059	− 3·72
12	1·000	+ ·998	+ ·132	+ ·066	+ 1·66
13	1·006	+ ·843	+ ·918	+ ·560	+ 2·49
14	1·000	+ ·665	+1·010	+ ·793	− 7·93

where $x = \delta\gamma$

$y = \delta K$

$z = K\,\delta e$

$$u = -\frac{K\mu}{(1-e^2)^{\frac{3}{2}}}\,\delta T$$

The normal equations from these observations are

$$\begin{array}{rrrrrl} 14·000x & + ·826y & + 3·399z & - 2·901u & - 6·70 & = 0 \\ & 8·397 & - 1·554 & + 1·652 & - 9·327 & = 0 \\ & & 7·329 & - 1·292 & - 5·153 & = 0 \\ & & & 5·234 & + ·168 & = 0 \end{array}$$

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*Contributions from Princeton Observatory, No. 3, p. 90.

Their solution gives

$x = + \cdot 1631$ or $\delta\gamma = +0 \cdot 16$ km.
$y = +1 \cdot 2825$ $\delta K = +1 \cdot 28$ km.
$z = + \cdot 8766$ $\delta e = + \cdot 0136$
$u = - \cdot 1298$ $\delta T = - \cdot 001$

and the final elements

e eccentricity	·0536	±	·0187
K semi amplitude	65·78	±	1·13
γ velocity system	−0·34	±	0·91
T time of periastron	0·019	±	·010
ω longitude of apse	90		

$a_b \sin i$ semi axis major = [6·40395] = 2,535,000 km.

$$\frac{m_f^3 \sin^3 i}{(m_b + m_f)^2} \quad [8 \cdot 91696] = \cdot 0826$$

This function of the masses, all that can be determined when the second spectrum cannot be seen, does not give us information in regard to the dimensions or masses of the system. From the photometric orbit we obtain the diameters of the individual stars in terms of the relative orbit, but the latter cannot be known without the ratio of the masses. As a first approximation Shapley assumed the masses equal and later used an empirical formula* for determining the masses of the components. This formula based on the relative light of the two bodies, has its constants determined from spectroscopic binaries in which both spectra show and the ratio of the masses are known. Using this formula $\mu_b = 0 \cdot 4 + 1 \cdot 2 L_b$, the ratio comes as brighter star 0·73 fainter star 0·27 total mass. If we use R S Vulpeculæ as an analogy where the system is somewhat similar, a dense bright small primary with a tenuous, large companion, it seems probable that the ratio of the masses assumed above is certainly not too great. With R S Vulpeculæ where the light of the brighter star is 0·526 of the system, the ratio of the masses is 3·20 while with T W Draconis, where the light of the brighter star is 0·885 of the system, it would certainly seem likely the ratio of masses would be greater. Nevertheless, the ratio given by the formula has been used, and I think may be safely considered as giving minimum dimensions to the orbit, but of course these dimensions depend upon this assumption and cannot be considered absolute, as in the two previous orbits. Using Shapley's two solutions, uniform and darkened, the inclinations are 75° 53′ and 79° 48′, while the semidiameters of brighter and fainter are ·130 and ·371 for the uniform and ·180 and ·334 for the darkened.

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*Contributions from the Princeton Observatory No. 3, p. 123

The dimensions of the system hence become

	Uniform	Darkened
a_b semi axis major primary orbit	2,614,000	2,575,000
a_f " " secondary "	7,067,000	6,963,000
a " " relative "	9,681,000	9,538,000
a in terms of radius of sun	13·92	13·71
a_b " "	3·76	3·70
r_b semi diam. brighter	1·81	2·47
r_f " fainter	5·10	4·58
mass brighter star	3·36	3·21
mass fainter "	1·24	1·19
total mass system	4·60	4·40
density brighter star	·57	·21
" fainter "	·0094	·0124

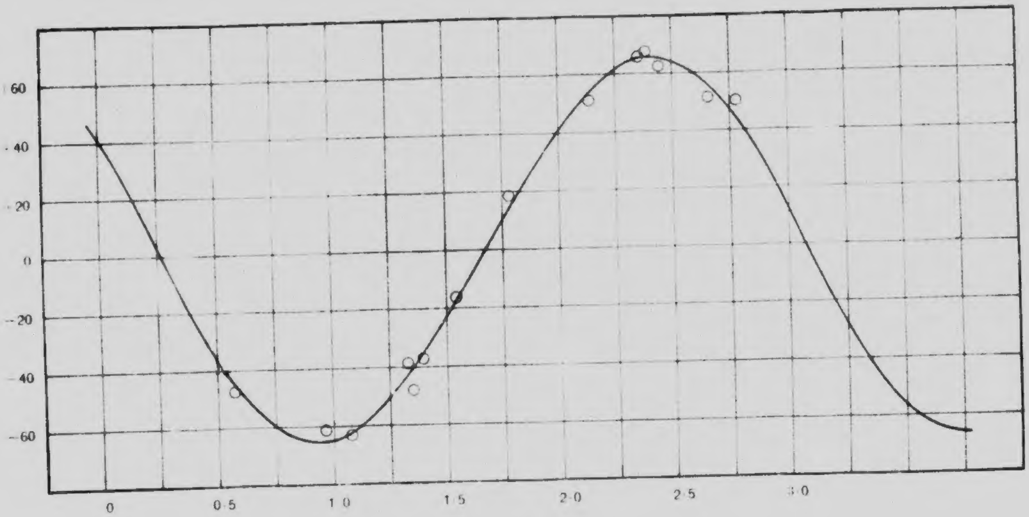

Fig. 3. T W Draconis

A graph of the velocity curve with the observations as circles is shown in Fig. 3.

CONCLUSION.

It may be of interest to summarize the dimensions of the two systems obtained here with the seven previously secured, that are known to me. The orbit of T W Draconis is not included in this summary, as owing to absence of the second spectrum the dimensions are not absolute. The following table gives the principal dimensions using the darkened photometric orbit in each case, where there are two or more solutions.

TABLE VI—ABSOLUTE DIMENSIONS ECLIPSING VARIABLES

Star	Greatest Radii		Masses		Densities		Distance of Centres.
	r_b	r_f	m_b	m_f	ρ_b	ρ_f	
β Aurigae[1]	2·81	2·81	2·40	2·36	0·11	0·11	17·7
U Herculis[1]	4·56	5·35	7·66	2·93	0·095	0·022	14·8
V Puppis[1]	8·45	7·70	19·4	19·4	0·042	0·055	12·7
B Lyrae[1]	16·2	40·6	1·42	14·2	0·0006	0·0004	59·9
RX Herculis[2]	1·54	1·38	0·89	0·89	0·25	0·34	7·5
W Ursae Majoris[3]	0·78	0·78	0·69	0·49	2·8	1·9	2·2
Z Herculis[4]	1·77	3·29	1·6	1·3	0·3	0·04	15·4
U Ophiuchi	3·23	3·23	5·36	4·71	0·18	0·16	12·8
R S Vulpeculae	2·05	10·25	5·40	1·69	0·63	0·0016	22·0

The above table is self explanatory if it is stated that the linear dimensions are in terms of the sun's radius and the masses and densities in terms of the mass and density of the sun. Fig. 4 gives a graphical representation of the dimensions of the systems of U Ophiuchi and R S Vulpeculæ and of the assumed dimensions of T W Draconis in terms of the sun. The intersections of the vertical and horizontal dotted lines represent the centers of mass of the systems. The series of figures to the left are the systems at primary eclipse and those to the right, a quarter revolution away, when at maximum separation. No attempt has been made to represent the stars as elliptical in form in this figure.

There is one other point in connection with these orbits, the coincidence or otherwise of the photometric and spectroscopic phases. Dr. Schlesinger has found in Algol, δ Libræ and U Herculis that the phase of mean primary eclipse comes slightly later, about an hour or so, than the time when u = 90° in the spectroscopic orbit, while it is evident the two should coincide. No explanation is offered for this discrepancy and it will be of interest to compare the results for the three variables discussed here.

In U Ophiuchi no attempt was made to apply a correction for θ, the phase in the circular orbit, owing to the high probable error and to the fact that nearly all the observations were near their maximum velocity, where they would have little weight in the determination of this correction.

In R S Vulpeculæ the velocity curve shows that the spectroscopic phase is considerably later than the photometric and, if the time when $u = 90°$ is computed, it is found to be $+0·128 \pm 0·010$ days. In T W Draconis $\omega = 90°$, therefore the time when $u = 90°$ is the time T found in the orbit, or $+0·019 \pm 0·010$ days. In both these cases, therefore, the spectroscopic phase is later than the photometric by about 3 hours in R S Vulpeculæ and 27 minutes in T W Draconis, in the opposite direction to that found by Schlesinger.

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[1] Astrophysical Journal 38, p. 169.

[2] " " 40, p. 399.

[3] " " 49, p. 189.

[4] " " 49, p. 192.

It is hard to imagine any physical cause which would account for a difference in phase in one direction in some instances and in the opposite direction in others, and though the differences appear to be considerably too great to be explained by errors of observation in either the spectroscopic or the photometric orbits, there seems to be no other alternative until more data are available.

Dominion Astrophysical Observatory,
Victoria, B.C.
August 6, 1919.

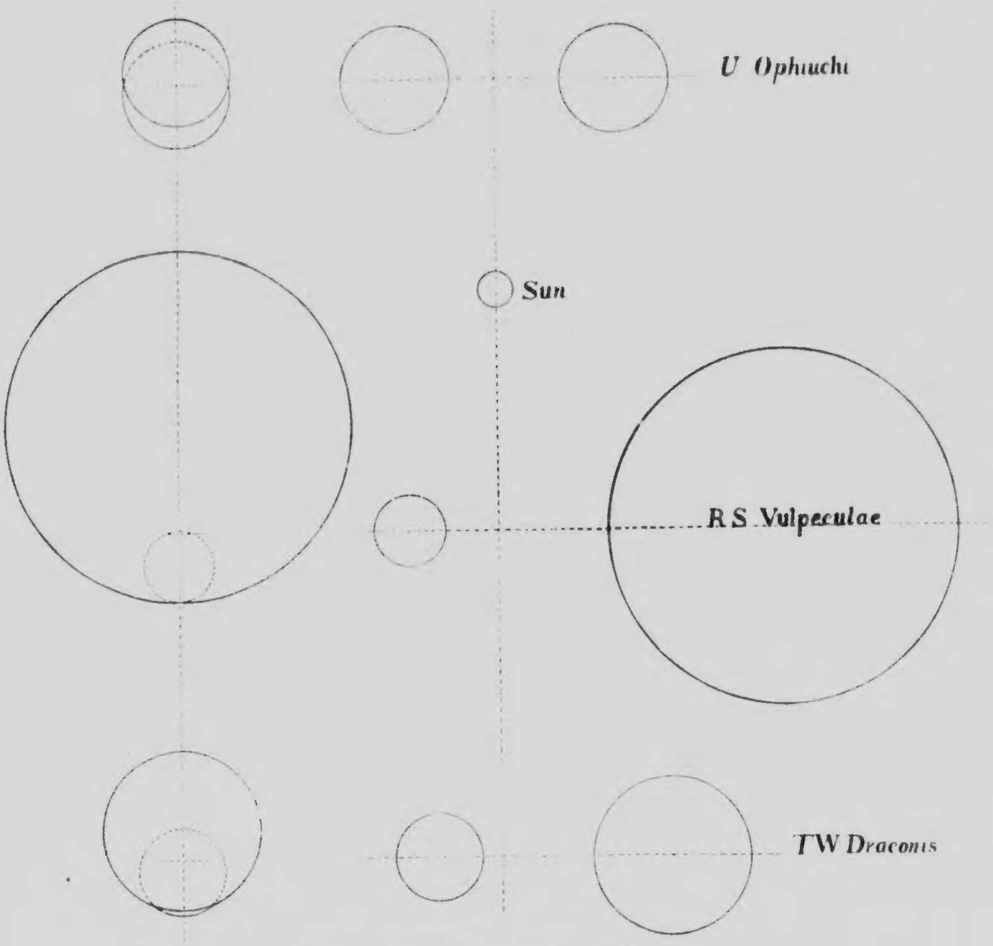

Fig. 4. Graphical Representation of Systems